EXERCISE – III

SUBJECTIVE QUESTIONS

1. The angle of a ΔABC are in A.P. and it is being given that b : c = $\sqrt{3}$: $\sqrt{2}$, then find $\angle$A.

2. In a triangle ABC, prove that for any angle θ,
b cos (A – θ) + a cos (B + θ) = c cos θ.

3. If cos A + cos B = 4 $\sin^2\left(\frac{C}{2}\right)$, prove that sides a, c, b of the triangle ABC are in A.P.

4. If in a ΔABC $\frac{\sin A}{\sin C} = \frac{\sin(A-B)}{\sin(B-C)}$, then prove that a^2, b^2, c^2 are in A.P.

5. Let a, b and c be the sides of ΔABC. If a^2, b^2 and c^2 are the roots of the equation $x^3 - Px^2 + Qx - R = 0$, where P, Q & R are constants, then find the value of $\frac{\cos A}{a} + \frac{\cos B}{b} + \frac{\cos C}{c}$, in terms of P, Q and R.

6. If the sides a, b, c of a triangle are in AP then find the value $\tan\frac{A}{2} + \tan\frac{C}{2}$ in terms of cot (B/2).

7. If D is the mid point of CA in triangle ABC and Δ is the area of triangle, then show that

$\tan(\angle ADB) = \frac{4\Delta}{a^2 - c^2}$.

8. If in a ΔABC, a = 6, b = 3 and cos (A – B) = 4/5 then find its area.

9. If in a triangle ABC $\angle$A = 30º and the area of triangle if $\frac{\sqrt{3}a^2}{4}$. then prove that either B = 4 C or C = 4 B.

10. Show that the radii of the three escribed circles of a triangle are roots of the equation,
$x^3 - x^2(4R + r) + x s^2 - r s^2 = 0$.

11. If in a triangle ABC, the altitude AM be the bisector of $\angle$BAD, where D is the mid point of side BC, then prove that $(b^2 - c^2) = a^2/2$

12. In a triangle ABC, if a tan A + b tan B $= (a + b)\tan\left(\frac{A+B}{2}\right)$ prove that triangle is isosceles.

13. In a Δ ABC, $\angle$C = 60º & $\angle$A = 75º. If D is a point on AC such that the area of the Δ BAD is $\sqrt{3}$ times the area of the Δ BCD find the $\angle$ ABD.

14. In triangle ABC, prove that the area of the incircle is to the area of triangle itself as,

$\pi : \cot\left(\frac{A}{2}\right) . \cot\left(\frac{B}{2}\right) . \cot\left(\frac{C}{2}\right)$.

15. DEF is the triangle formed by joining the points of contact of the incircle with the sides of the triangle ABC : prove that

(i) its sides are $2r\cos\frac{A}{2}$, $2r\cos\frac{B}{2}$, and $2r\cos\frac{C}{2}$,

(ii) its angles are $\frac{\pi}{2} - \frac{A}{2}$, $\frac{\pi}{2} - \frac{B}{2}$, and $\frac{\pi}{2} - \frac{C}{2}$

(iii) its area is $\frac{2\Delta^3}{abcs}$, i.e. $\frac{1}{2}\frac{r}{R}\Delta$.

16. If the circumcentre of the Δ ABC lies on its incircle then prove that, cos A + cos B + cos C = $\sqrt{2}$

17. A cyclic quadrilateral ABCD of area $\frac{3\sqrt{3}}{4}$ is inscribed a unit circle. If one of its sides AB = 1 & the diagonal BD = $\sqrt{3}$, find lengths of the other sides.

18. Perpendicular are drawn from the angles A, B, C of an acute anlged triangle on the opposite sides and produced to meet the circumscribing circle. If these produced parts be α, β, γ respectively, show that

$$\frac{a}{\alpha} + \frac{b}{\beta} + \frac{c}{\gamma} = 2\,(\tan A + \tan B + \tan C)$$

19. The product of the sines of the angles of a triangle is p and the product of their cosines is q. Show that the tangents of the angles are the roots of the equation $qx^3 - px^2 + (1 + q)\,x - p = 0$

Prove that :

20. $\dfrac{b-c}{r_1} + \dfrac{c-a}{r_2} + \dfrac{a-b}{r_3} = 0$

21. $a \cot A + b \cot B + c \cot C = 2(R + r)$

22. $\dfrac{r_1}{(s-b)(s-c)} + \dfrac{r_2}{(s-c)(s-a)} + \dfrac{r_3}{(s-a)(s-b)} = \dfrac{3}{r}$

23. $\dfrac{r_1 - r}{a} + \dfrac{r_2 - r}{b} = \dfrac{c}{r_3}$

24. $\dfrac{abc}{s} \cos\dfrac{A}{2} \cos\dfrac{B}{2} \cos\dfrac{C}{2} = \Delta$

25. $(r_1 + r_2) \tan\dfrac{C}{2} = (r_3 - r) \cot\dfrac{C}{2} = c$